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# Analogue gravity in photon fluids

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We discuss the main features and our ongoing of work in the field on analogue gravity in photon fluids.

*Keywords*: Nonlinear optics; general relativity.

## 1. Introduction

In the recent decades, analogue gravity models have become an ever increasing research field since they allow to experimentally test and study concepts of general relativity and quantum field theory. It dates back to 1981, when Unruh[1] unravelled that the propagation of sound modes in a flowing fluid behave exactly as that of a massless scalar field in a curved space time metric. Hence the idea of those analogue models is that the modification of the propagation of a wave (i.e. electromagnetic wave) in a curved space-time can be reproduced by an analogue wave travelling in a background material with space and time dependent properties. Up to today, many systems have been proposed and developed to mimic space-time analogues, such as liquid Helium[2], Bose-Einstein Condensates (BECs)[3,4], surface waves on water[5] and nonlinear optics[6–8]. Many of these systems involve complex experimental challenges that make measurements of in situ parameters extremely difficult, or physical limitations that restrain the accessibility to phenomena such as analogue Hawking radiation. It is therefore an ever ongoing motivation to find new analogue systems that address these challenges.

Here we introduce a new approach with a so called Photon fluid in a propagating geometry. The photons of a laser beam propagating through a defocusing, nonlinear medium, can be described as weakly interacting gas of particles. The optical field plays the role of the order parameter similar to that in the Gross-Pitaevski formalism for superfluids. It therefore bears strong resemblance to BECs and as we will show has under specific circumstances superfluid characteristics. Essentially, the transverse beam profile is the analogue of a fluid surface, whereas the propagation direction maps into a time coordinate. The flow and the density of the fluid can be controlled by the topological phase and intensity of the laser beam. It is this property that provides an easy way to mimic radial and/or rotational flow scenarios to create horizons and ergoregions as seen for a Schwarzschild or Kerr black hole[9,10].

## 2. Photon fluid

The propagation of a monochromatic laser beam with a vacuum optical wavelength $\lambda$ in a self-defocusing medium can be described at steady state by the non-linear Schrödinger equation (NLSE):

$$\partial_z E = \frac{i}{2k}\nabla^2 E - i\frac{kn_2}{n_0}|E|^2 E \tag{1}$$

where $k = 2\pi n_0/\lambda$ is the optical wavenumber, $\lambda$ is the vacuum wavelength of the light, $n_0$ is the linear and $n_2$ the nonlinear refractive index, respectively. By considering the propagation direction $z$ as time coordinate $t = zn_0/c$, where $c$ is the speed of light in vacuum, and the electric field $E(\mathbf{r}, t) = \sqrt{\rho(\mathbf{r}, t)}\exp(i\phi(\mathbf{r}, t))$ as a function of fluid density $\rho$ and phase $\phi$, one arrives at a set of hydrodynamical equations that describes the laser beam as a 2+1-dimensional quantum fluid of light[11,12],

$$\partial_\tau \rho + \nabla(\rho \boldsymbol{v}) = 0 \tag{2}$$

$$\partial_\tau \psi + \frac{1}{2}v^2 + \frac{c^2 n_2}{n_0^3}\rho - \frac{c^2}{2k^2 n_0^2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} = 0. \tag{3}$$

The gradient of the transverse beam phase determines the fluid flow velocity $\boldsymbol{v} = (c/kn_0)\boldsymbol{\nabla}\phi = \boldsymbol{\nabla}\psi$ and the speed of long-wavelength sonic waves is given by $c_s = \sqrt{c^2|n_2|\rho/n_0^3}$.

Transverse wave perturbations propagating on an intense background beam obey the well known Bogoliubov dispersion relation[11,12]. Considering nonlocal nonlinearities, an extension of the Bogoliubov theory leads to a modified form of the dispersion relation[13],

$$(\Omega - vK)^2 = \frac{c^2|n_2|\rho}{n_0^3}\hat{R}(K, n_0\Omega/c)K^2 + \frac{c^2}{4k^2 n_0^2}K^4, \tag{4}$$

relating frequency $\Omega$ and wavenumber $K$ of phonon excitations. Here $\hat{R}$ is the Fourier transform of the medium response function that can be modelled as a Lorentzian function with width $\sigma_L$[14]. For excitations with wavenumbers smaller

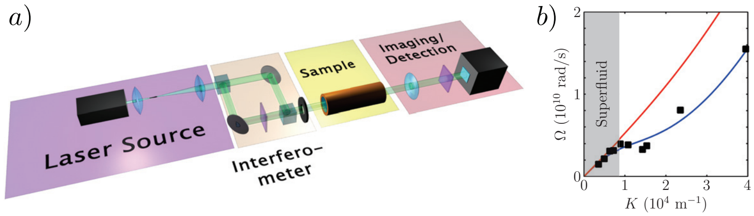


Fig. 1. a) Experimental layout used for measuring the dispersion relation. A collimated, flat beam is launched through a cylindrical sample filled with a methanol/graphene solution and is imaged by a camera/lens system that can be translated along the propagation direction of the beam. b) Bogoliubov dispersion relation showing the phonon part for elementary excitations. Bold red line: local nonlinearity, bold blue line: nonlocal nonlinearity, black squares: experimental data.

than the inverse of the nonlocal length ($K < 1/\sigma_{\rm L}$) the dispersion is a linear function in $K$ ($\Omega \approx c_s K$). This is of particular relevance, since only a linear dispersion guarantees superfluid behaviour[15] and, in the context of analogue gravity, Lorentz invariance in the acoustic metric[9,16].

We measured the dispersion relation with a technique traditionally used in oceanography, where the spatiotemporal evolution of surface waves that naturally occur on top of the background fluid, in our case, small intensity fluctuations on the beam profile, is recorded[17]. Then by calculating the Fourier transform, one obtains the dispersion relation $\Omega(K)$ (Fig. 2). The main finding of this result is a purely parabolic dispersion that shows the single particle part ($\Omega \propto K^2$) of elementary excitations. The phonon part ($\Omega \propto K$) was measured with different technique, where instead of using waves that naturally exist in the fluid, we are seeding a specific wavelength with a second weak beam and measure its phase velocity ($v_{ph} = \Omega/K$). Using the parameters and the technique described in Ref.[17], we indeed show that these follow the dispersion given by Eq. (4). Hence there exists a critical velocity $v_c$ for which a fluid with flow speed $v < v_c$ behaves as a superfluid (Fig. 1b).

## 3. Emergent geometries

The general idea in analogue gravity models is that the propagation of a scalar field in a curved spacetime is the same as that of small amplitude excitations on a

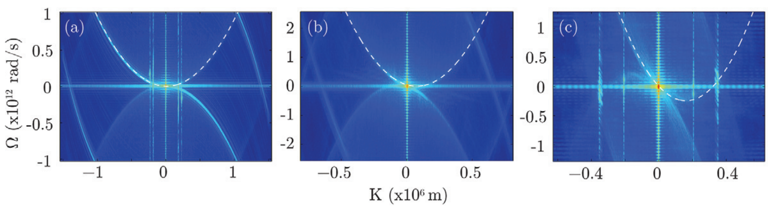


Fig. 2. Photon fluid dispersion relation as measured with oceanographic technique. The flow $v$ of the effective medium can be controlled by the phase of the background field: (a) $v_{bg} = 0$ m/s, (b) $v_{bg} = 1.3 \times 10^6$ m/s, and (c) $v_{bg} = 3.0 \times 10^6$ m/s.

background fluid with an inhomogeneous flow configuration. These models are generally based on perturbative schemes, i.e. the nonlinear problem is solved for small amplitude perturbations that propagate on a background solution. As such, the geometrical analogy holds only for the linearised fluctuation whereas the background is not affected by the perturbation dynamics. However, when the fluctuations cannot be described as a background perturbation anymore, the nonlinear dynamics of this scalar field can be described as an emergent geometry, which is generated by the field itself[18]. Hence, emergent geometries provide valuable insights into the dynamical interplay between the scalar field and the metric.

In this work, the nonlinear acoustic disturbances are realised in a photon fluid as described earlier. Considering a 1D-system and by applying the $\partial x$ operator to equation 3 and neglecting the quantum pressure term, we arrive at:

$$\partial_t v + v\partial_x v + \frac{\partial P}{\rho} = 0. \tag{5}$$

Together with Eq. (2) and (3), these equations represent the Navier-Stokes equations in a 1D compressible, non-viscous fluid. The nonlinear term, $v\partial_x v$ determines the steepening rate of a wave and will eventually lead to shock formation[19]. The connection between the nonlinear fluid equations and a dynamic acoustic metric can be shown by writing these equations in terms of Riemann invariants $\psi_\pm = v \pm 2c_s$, where $c_s$ is the local speed of sound and we obtain two advection equations

$$\partial_t \psi_\pm + c_\pm \partial_x(\psi_\pm) = 0 \tag{6}$$

where $c_\pm = 3/4\psi_\pm + 1/4\psi_\pm = v \pm cs$. These equations simply state that the two variables $\psi_\pm$ are conserved along the characteristics curves defined by $dx/dt \equiv c_\pm$. Therefore, in a nonlinear wave the constant quantities $\psi_\pm$ are propagated in spacetime at the characteristic speeds $c_\pm = v \pm c_s$, so they travel at the speed of sound relative to the flowing fluid. As in linear acoustic analogues, the characteristic curves demarcate the region of causally connected events. The acoustic line element can be written as

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = (dx - (v + c_s)dt)(dx - (v - c_s)dt) \tag{7}$$

and thus the characteristics of the fluid equations coincide with the null geodesic defined by the acoustic metric $g_{\mu\nu}$. In the experiment, we use a setup similar to that described in[17] to produce an interference pattern between a strong pump beam and a weaker probe beam to create a density wave in the photon fluid. This is then launched through a 21cm long sample filled with a methanol/graphene solution and the profile is imaged at the output (Fig.(3a)). The density wave is shown in Fig.(3). For low powers (quasi-linear regime) the shape is sinusoidal, whereas for high powers (nonlinear regime) a clear self-steepening effect is observed. In Fig.(3) we plot the trajectories of points with the same density $\rho$ for the nonlinear case and show the convergence of this characteristic as a clear indication of an increasing curvature of space-time.

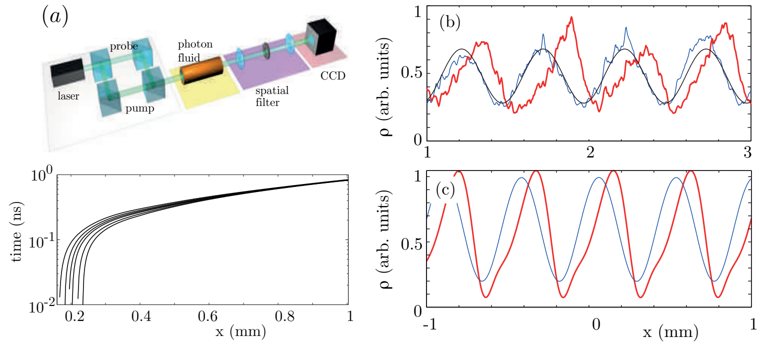


Fig. 3. (a) top: Experimental setup - bottom: Numerical results. Trajectories of points with the same density generated by the right-moving high amplitude density wave shown in Fig.(3c). (b) measured density wave for low power (blue line) and high power (red line). (c) Numerical results under the same conditions of the experiment.

## 4. Conclusions

The results shown here illustrate that using a gas of photons it is possible to create an effective flowing fluid that exhibits the characteristic traits required for analogue gravity studies. A first result shows that we can use these photon fluids to study the natural emergence of geometry in the presence of perturbations or oscillations that affect the background spacetime metric. Notice that, from a different perspective, the experimental platform discussed here has been used to evidence the emergence of large scale incoherent structures, whose collective global behaviour has been described by a long-range Vlasov formalism completely analogous to that used to describe the formation and the interaction of galaxies in the Universe[20]. Future work will be directed at creating and studying artificial horizons in these photon fluids with the possibility of extending current studies from 1D to 2D geometries where, for example, rotation may also be included in the background metric.

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